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Reg. No. : .....

Name : .....

Fifth Semester B.Sc. Degree Examination, December 2017
First Degree Programme Under CBCSS
MATHEMATICS
Core Course
MM 1541 : Real Analysis – I
(2013 Admn.)

Time : 3 Hours Max. Marks : 80

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SECTION – I

All the **first 10** questions are **compulsory**. They carry **1** mark **each**.

1. If $a.b = 0$; $a, b \in \mathbb{R}$, prove that either $a = 0$ or $b = 0$.
2. State Bernoulli's inequality.
3. State triangle inequality of real numbers.
4. Define convergence of a sequence of real numbers.
5. Give a bounded sequence that is not a Cauchy sequence.
6. State Monotone subsequence theorem.
7. Define a contractive sequence.
8. State the Cauchy criterion for the convergence of a series.
9. Find the cluster points of the set $A = (0, 2)$.
10. Define left hand limit of a function.

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SECTION – II

Answer **any 8** questions among the questions **11** to **22**. These questions carry **2** marks **each**.

11. Determine the set A of all real numbers x such that $x^2 + x > 2$.

12. If $a \in \mathbb{R}$ is such that $0 \leq a < \varepsilon$ for every $\varepsilon > 0$, then $a = 0$.

13. Prove that if $c \geq 0$, then $|a| \leq c$ if and only if $-c \leq a \leq c$; $a, b, c \in \mathbb{R}$.

14. Let $a \in \mathbb{R}$. If x belongs to the neighbourhood $V_\varepsilon(x)$ for every $\varepsilon > 0$, then $x = a$.

15. If $X = (x_n)$ is a convergent sequence of real numbers and if $x_n \geq 0$ for all $n \in \mathbb{N}$, then prove that $x = \lim (x_n) \geq 0$.

16. Prove that the sequence $((-1)^n)$ is not convergent.

17. Give an example of two divergent sequences X and Y such that their product X.Y converges.

18. Prove that $\left(\frac{1}{n}\right)$ is a Cauchy sequence.

19. Prove that every convergent sequence of real numbers is a Cauchy sequence.

20. If the series $\sum x_n$ converges, prove that $\lim x_n = 0$.

21. Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$ converges.

22. Prove that $\lim_{x \to 0} \frac{1}{x}$ does not exist.

SECTION – III

Answer **any 6** questions among the questions **23** to **31**. These questions carry **4** marks **each**.

23. State Algebraic properties of real numbers.

24. If x and y are real numbers with $x < y$, prove that there is a rational number r such that $x < r < y$.

25. If $I_n = [a_n, b_n]$; $n \in \mathbb{N}$ is a nested sequence of closed bounded intervals, prove that there exists a real number ξ such that $\xi \in I_n$ for all $n \in \mathbb{N}$.

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26. Prove that the sequence (n) is divergent.

27. Let $X = (x_n)$ be a sequence of real numbers that converges to x and suppose $x_n \geq 0$. Prove that the sequence $\left(\sqrt{x_n}\right)$ of positive square roots converges and $\lim\left(\sqrt{x_n}\right) = \sqrt{x}$.

28. State and prove Bolzano-Weierstrass Theorem.

29. Let (x_n) and (y_n) be sequences of real numbers such that $x_n \leq y_n \ \forall n \in \mathbb{N}$. Prove that if $\lim(x_n) = +\infty$, then $\lim(y_n) = +\infty$.

30. Let $f : A \to \mathbb{R}$ be a function and c a cluster point of A. Prove that f can have only one limit at c.

31. Show that $\lim_{x \to 0} \sin\frac{1}{x}$ does not exist.

SECTION – IV

Answer **any 2** questions among the questions **32** to **35**. These questions carry **15** marks **each**.

32. Prove the following :

a) If $x \in \mathbb{R}$, then there exists $n_x \in \mathbb{N}$ such that $x < n_x$.

b) If $S = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$, then inf S = 0.

c) If $t > 0$, there exists $n_t \in \mathbb{N}$ such that $0 < \frac{1}{n_t} < t$.

33. a) State and prove squeeze theorem for sequences.

b) State and prove Monotone Convergence Theorem.

34. a) Let $p > 0$. Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if and only if $p > 1$.

b) State and prove Raabe's Test.

35. a) Prove that a number $c \in \mathbb{R}$ is a cluster point of a subset A of $\mathbb{R}$ if and only if there exists a sequence (a_n) in A such that $\lim(a_n) = c$ and $a_n \neq c \ \forall n \in \mathbb{N}$.

b) State and prove the sequential criterion for limits of a function.

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